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REPRESENTATION OF PROPAGATION PARAMETERS FOR THE PLASMA IN A MAGNETIC FIELD

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REPRESENTATION OF PROPAGATION PARAMETERS FOR THE PLASMA IN A MAGNETIC FIELD.

In a recent paper the properties of a uniform plasma for the propagation of electromagnetic waves were represented by curves plotted in the complex propagation plane[1]. The effect of a d.c. magnetic field can be shown by extending these curves to three dimensional models in the way described below.

The complex propagation plane is represented by coordinates $A = \alpha\ (c/\omega)$ and $B = \beta\ (c/\omega)$ where α and β are the real and imaginary parts of the complex propagation constant and (c/ω) is the ratio of the velocity of light to the frequency of propagation. In reference 1 the parameters plotted were the normalized collision frequency $Z = \nu/\omega$, where ν is the collision frequency and the normalized electron density $X = (\omega_p/\omega)^2$ where ω_p is the plasma frequency. The equations for the curves were derived from a conformal transformation of corresponding curves in the complex dielectric plane. These equations in turn were derived from the wave equation using the dielectric coefficient for a uniform plasma.

A later communication[2] indicated that by suitably choosing the normalized parameters to include the electron cyclotron frequency ω_b these same curves could be used for the case of a plasma in the presence of a dc magnetic field. The purpose of this note is to point out that the original curves plotted in the complex propagation plane can be extended to three-dimensional models for the case of a dc magnetic field by introducing a third coordinate $Y = \omega_b/\omega$.

Appleton's equation for the complex refractive index of a magneto-ionic medium is given by[3]

$$n^2 = 1 - \cfrac{X}{1 - jZ - 1/2\,\cfrac{Y_y^2}{1 - X - jZ} \pm \left[1/4\,\cfrac{Y_y^4}{(1 - X - jZ)^2} + Y_z^2\right]^{1/2}} \qquad (1)$$

where X and Z are the normalized parameters defined above and where

$Y_z = \omega_z/\omega$ = the normalized cyclotron frequency in the direction of propagation, z

$Y_y = \omega_y/\omega$ = the normalized cyclotron frequency in the y-direction

$\omega_z = (e/m)B \cos \theta$

$\omega_y = (e/m)B \sin \theta$

θ = angle between the magnetic field B and the direction of propagation, z

For the case of a longitudinal magnetic field, $Y_y = 0$, $Y_z = Y$ and (1) reduces to

$$n^2 = 1 - \frac{X}{1 - jZ \pm Y} \tag{2}$$

Since the square of the refractive index is equal to the dielectric coefficient, one can rationalize the right-hand side of (2) and equate the real and imaginary parts to K_r and K_i, the real and imaginary parts of the dielectric coefficient K. A conformal transformation from the complex dielectric plane to the complex propagation plane yields the following two equations for the normalized collision frequency case and for the normalized electron density case.

$$B^2 - A^2 + \frac{2AB(1 \pm Y)}{Z} = 1 \tag{3}$$

$$(A^2 + B^2)^2 - \left(2 - \frac{X}{1 \pm Y}\right)(B^2 - A^2) + \left(1 - \frac{X}{1 \pm Y}\right) = 0 \tag{4}$$

For the special case of no magnetic field these equations reduce to Equations (12b) and (13b) of reference 1. They are also consistent with the redefined parameters of reference 2. Plots of equations (3) and (4) are shown in Figs. 1, 4 where surfaces are formed for different values of Z and X. The plus sign in (3) and (4) corresponds to propagation of the ordinary wave while the minus sign corresponds to propagation of the extraordinary wave.

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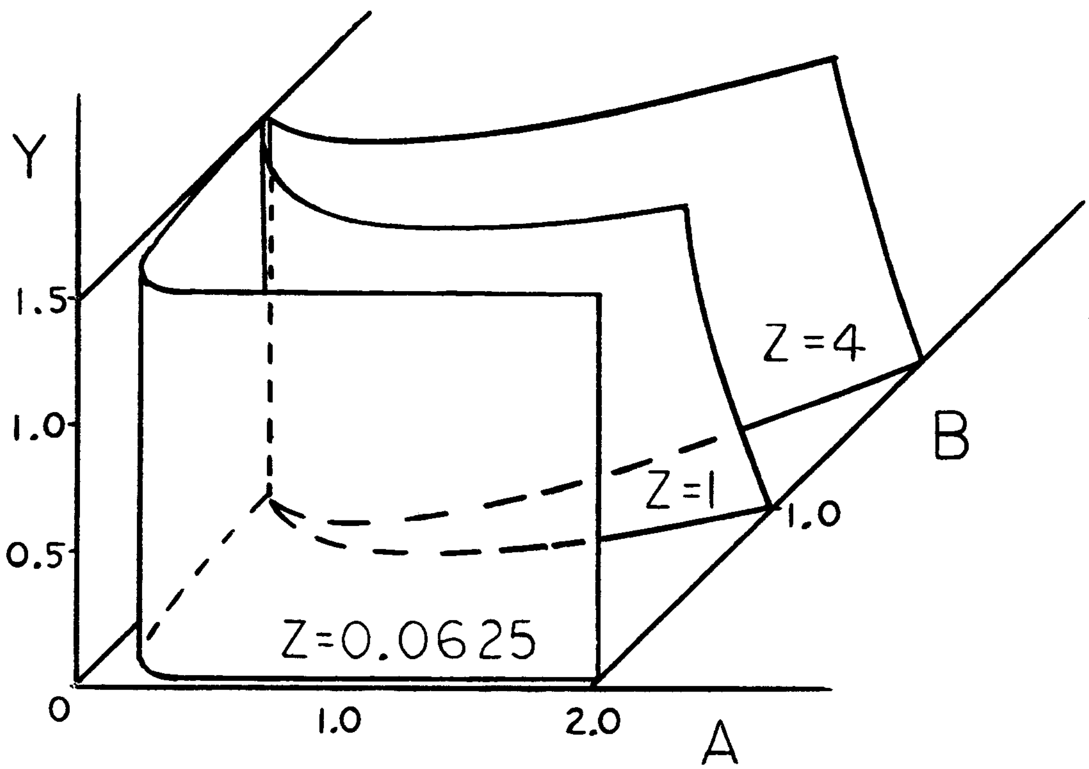

FIGURE 1

COLLISION FREQUENCY MODEL
ORDINARY WAVE

$A = (c/\omega)\alpha$ = normalized attenuation constant
$B = (c/\omega)\beta$ = normalized phase shift contant
$Y = \omega_z/\omega$ = normalized cyclotron frequency

$Z = \nu/\omega$ = normalized collision frequency

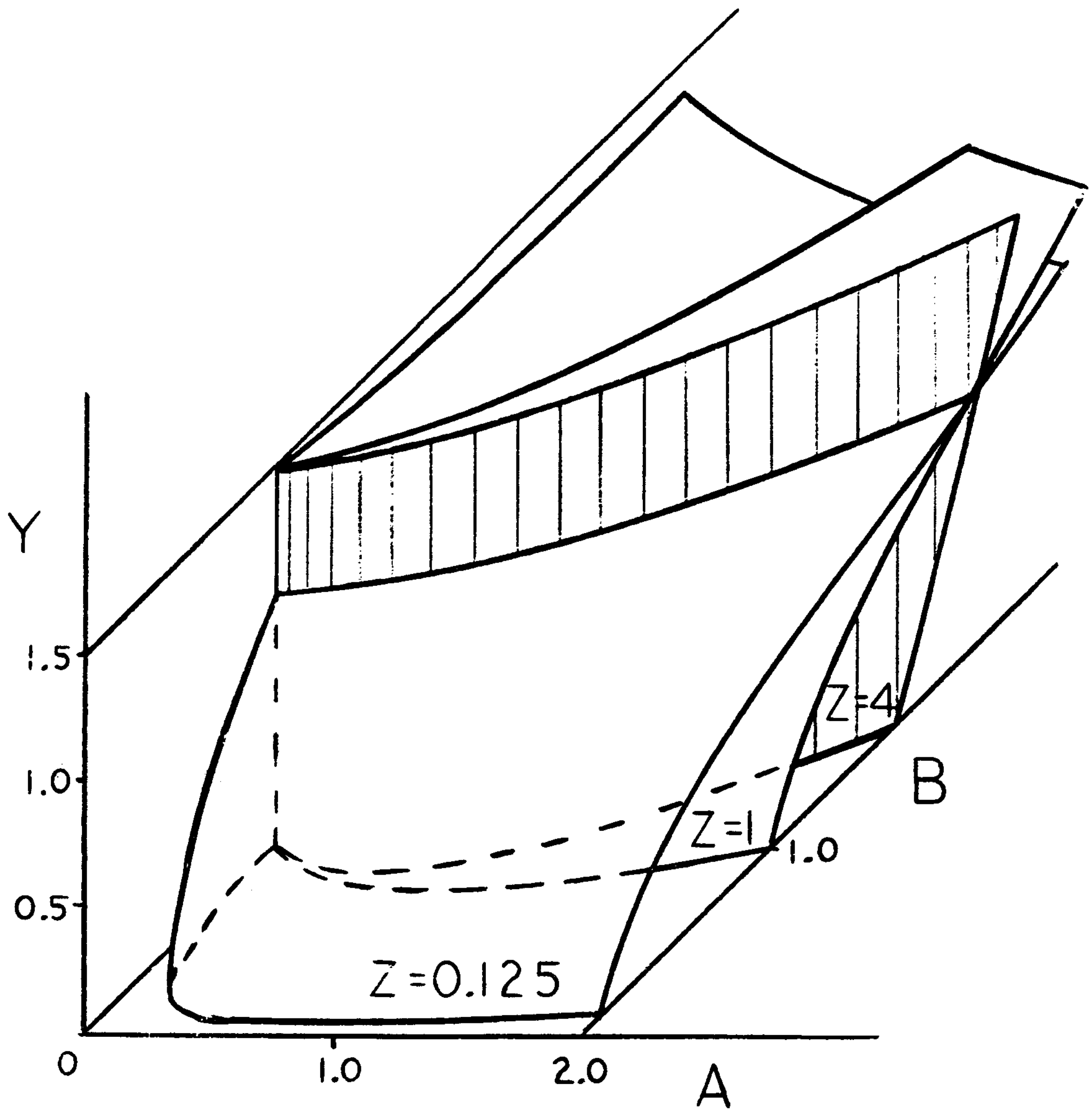

FIGURE 2

COLLISION FREQUENCY MODEL
EXTRAORDINARY WAVE

$A = (c/\omega)\alpha$ = normalized attenuation constant
$B = (c/\omega)\beta$ = normalized phase shift constant
$Y = \omega_z/\omega$ = normalized cyclotron frequency

$Z = \nu/\omega$ = normalized collision frequency

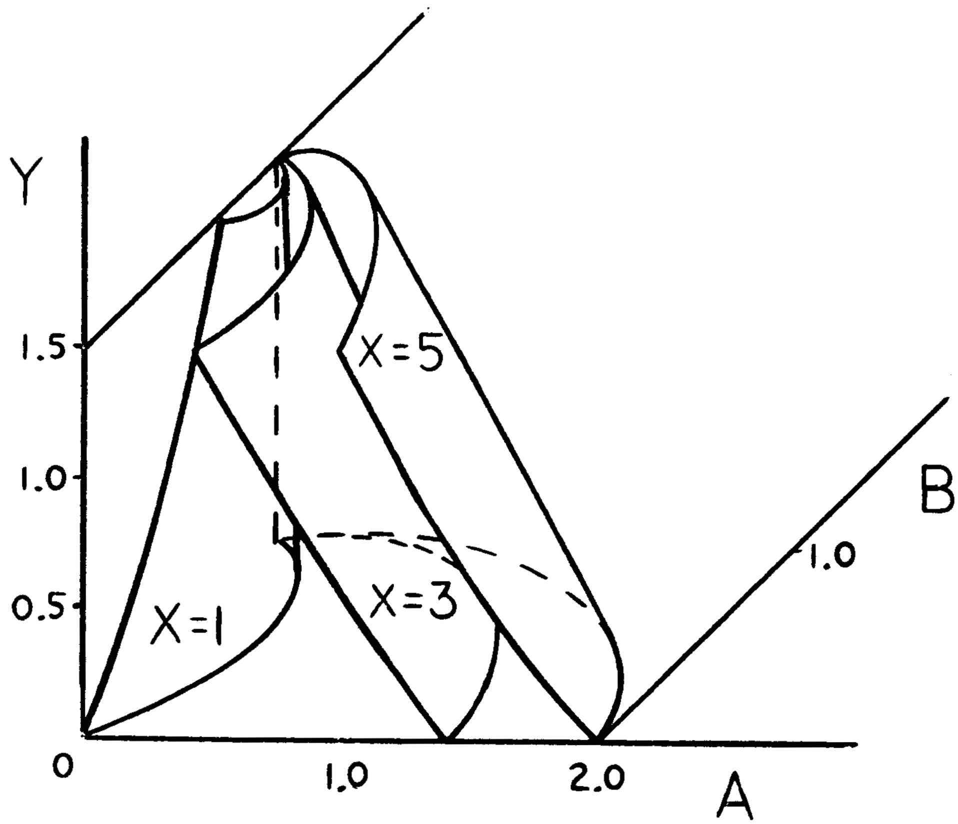

FIGURE 3

ELECTRON DENSITY MODEL
ORDINARY WAVE

$A = (c/\omega)\alpha$ = normalized attenuation constant
$B = (c/\omega)\beta$ = normalized phase shift constant
$Y = \omega_z/\omega$ = normalized cyclotron frequency

$X = (\omega_p/\omega)^2$ = normalized electron density

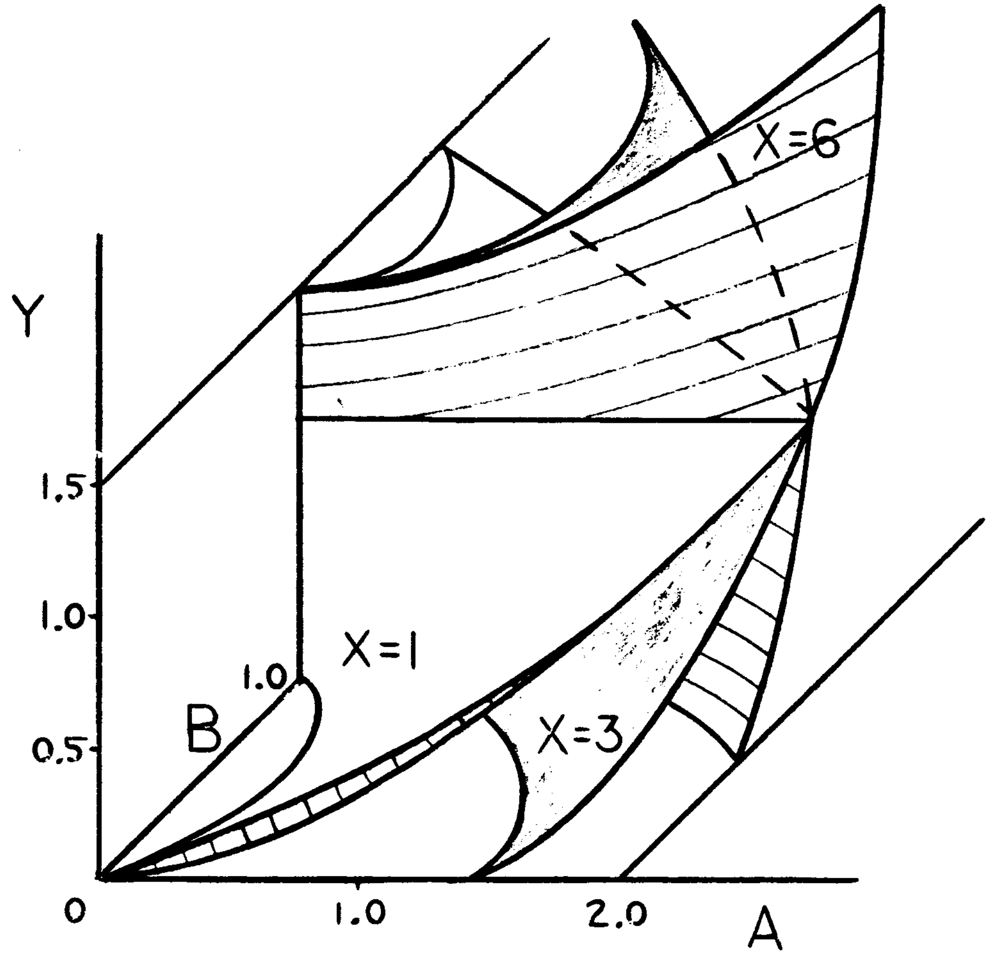

FIGURE 4

ELECTRON DENSITY MODEL
EXTRAORDINARY WAVE

A = $(c/\omega)\alpha$ = normalized attenuation constant
B = $(c/\omega)\beta$ = normalized phase shift constant
Y = ω_z/ω = normalized cyclotron frequency
X = $(\omega_p/\omega)^2$ = normalized electron density